EPFL

L2 –Hydraulic turbomachines and energy conversion

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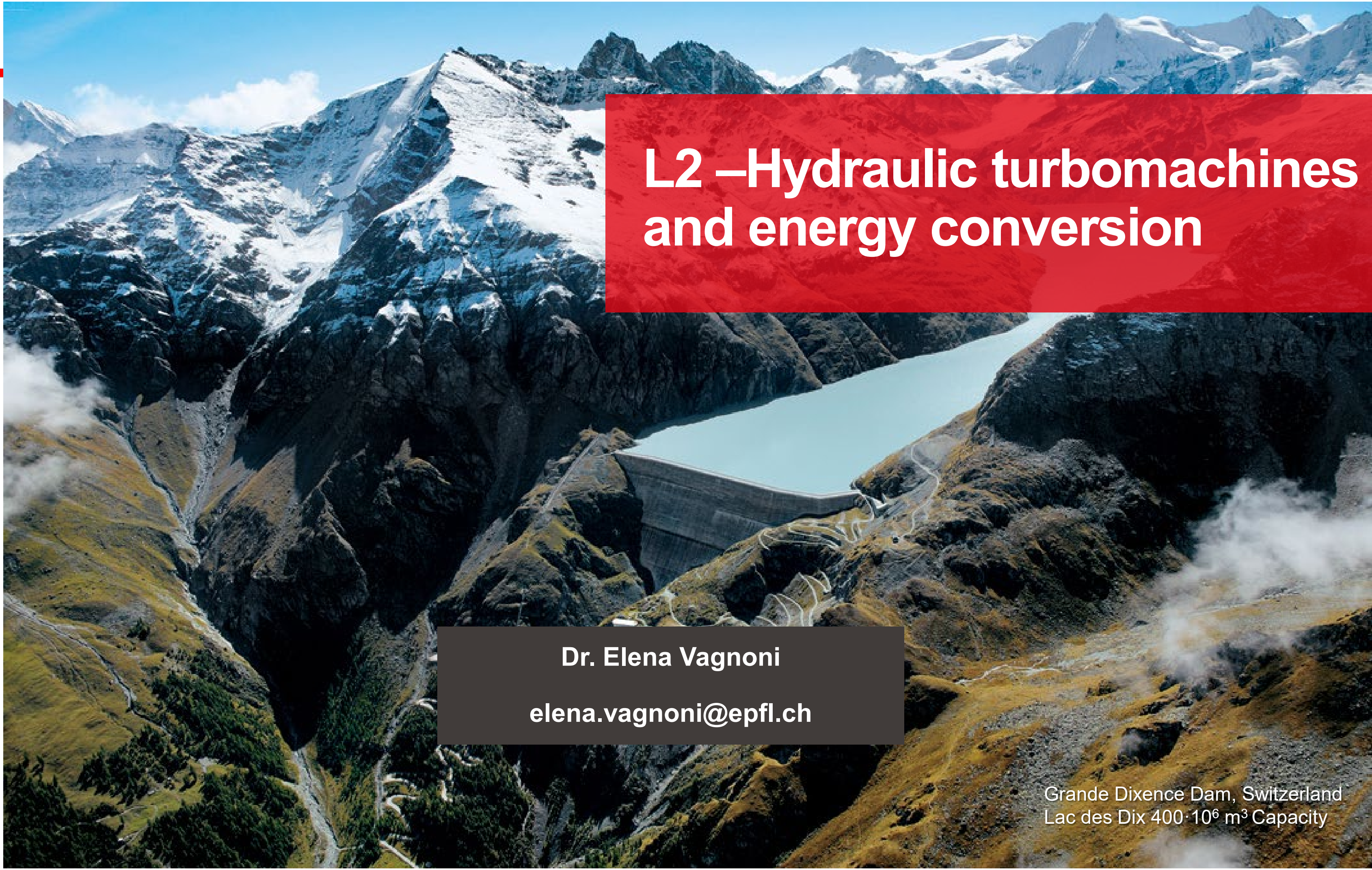


Grande Dixence Dam, Switzerland
Lac des Dix 400·10^6 m^3 Capacity

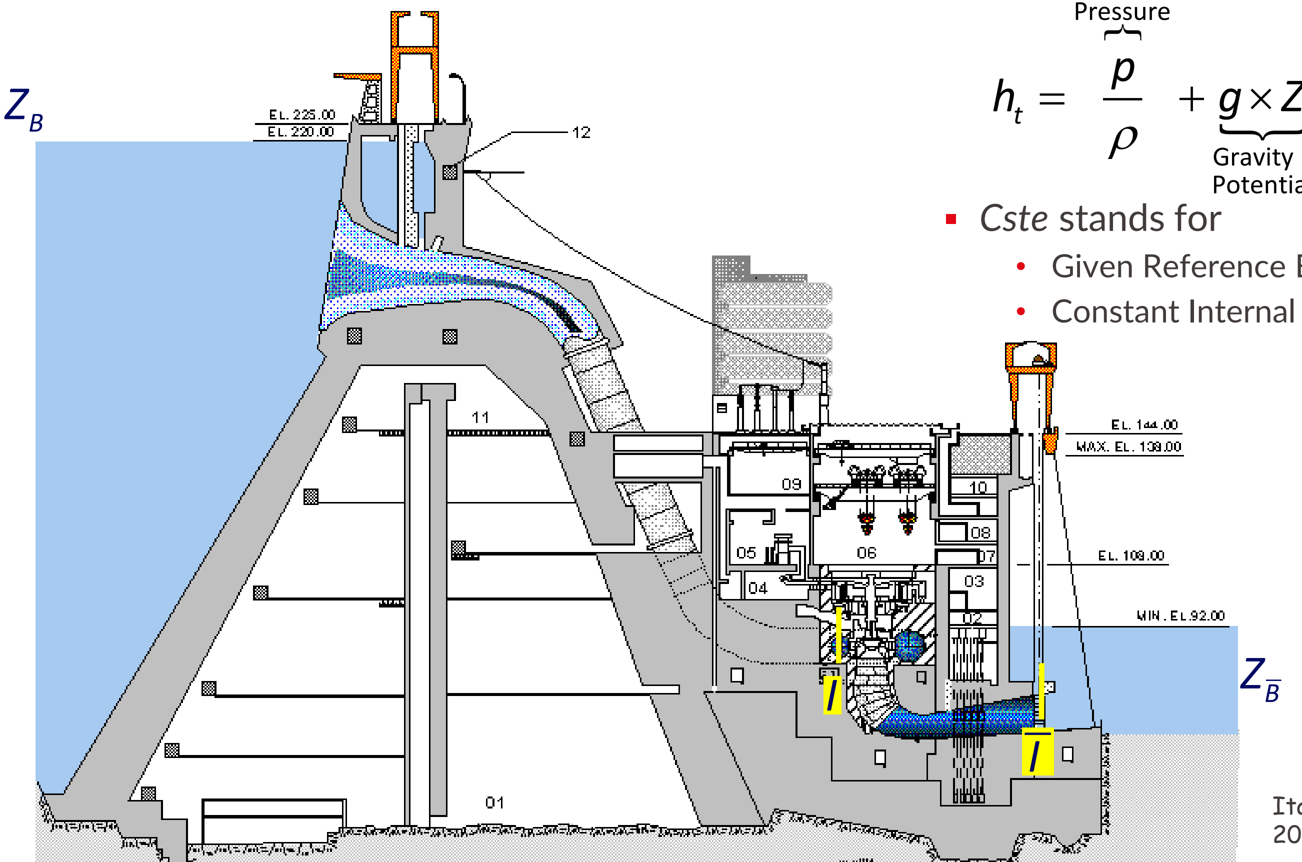
École
polytechnique
fédérale
de Lausanne

Topics of the lecture

- Specific Energy Conversion
- Classification of hydraulic turbomachines
  - Types
  - Performance

From L1: Specific Hydraulic Energy

Specific Energy Balance



- Local Mean Flow Specific Energy: Flow Property

$$h_t = \overbrace{\frac{p}{\rho}}^{\text{Pressure}} + \underbrace{g \times Z}_{\text{Gravity Potential}} + \frac{\overbrace{\vec{C}^2}^{\textit{Kinetic}}}{2} + Cste \quad \left(\text{J}\cdot\text{kg}^{-1}\right)$$

- *Cste* stands for
  - Given Reference Elevation Z_{ref} (e.g. Sea Level)
  - Constant Internal Specific Energy

Available Specific Hydraulic Energy

$$E \triangleq gH_I - gH_{\bar{I}}$$
$$= g\left(Z_B - Z_{\bar{B}}\right) - \sum gH_r^T$$

Itaipú, Brazil/Paraguay
20 x 740 MW Francis

Specific Hydraulic Energy

Hydraulic System: Specific Energy and Discharge Balance

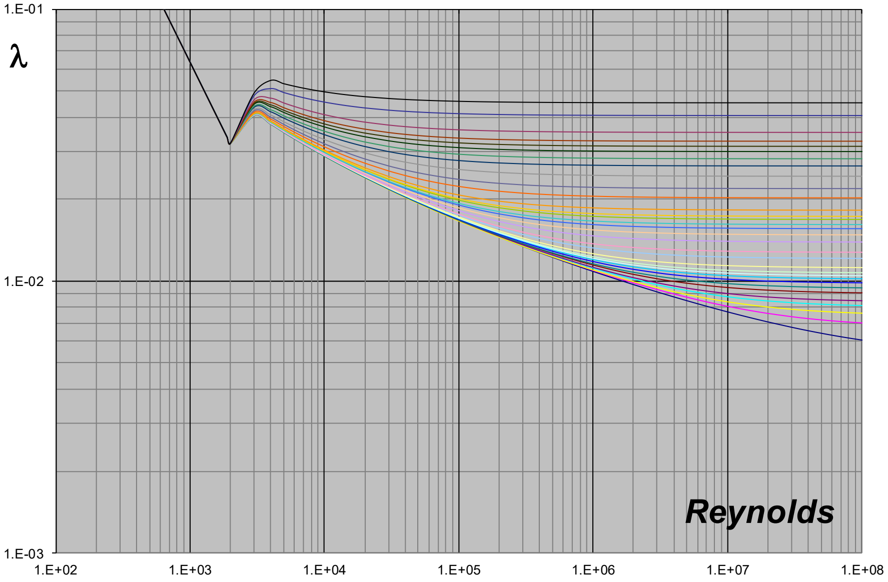
- Specific Energy Balance $gH_1 = gH_2 + \sum_{1\div 2} (gH_r)_i$
  - Steady Flow
  - Straight Stream Lines, free of Secondary Flow

- Discharge Balance $Q_1 = Q_2$
  - Mass conservation $A_1 \times C_1 = A_2 \times C_2$

Specific Hydraulic Energy

Specific Energy Losses : The Pipe Distributed Losses

$$gH_{r1\div2} = \lambda\left(Re;\frac{k}{D_h}\right)\frac{L_{\widehat{12}}}{D_h}\frac{\vec{C}^2}{2} \quad \left(\mathrm{J\cdot kg^{-1}}\right)$$



- λ Local Loss Coefficient $\lambda = \lambda\left(\mathrm{Re},\frac{k}{D_h}\right)$
- Re Reynolds Number $\mathrm{Re} = \frac{CD_h}{\nu}$
- k Roughness
- D_h Hydraulic Diameter $D_h = \frac{4A}{P_{wet}}$

Specific Hydraulic Energy

Specific Energy Losses : The Pipe Distributed Losses

Churchill Empirical Formula

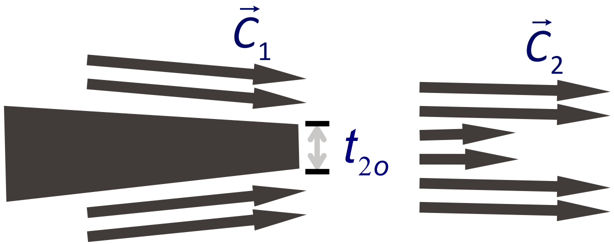
$$\lambda\left(\mathrm{Re};\frac{k_s}{D}\right)=8\left[\left(\frac{8}{\mathrm{Re}}\right)^{12}+\frac{1}{(A+B)^{\frac{3}{2}}}\right]^{\frac{1}{12}}$$

$$\text{with}\quad A=\left[2.457\cdot\ln\frac{1}{\left(\frac{7}{\mathrm{Re}}\right)^{0.9}+0.27\cdot\frac{k_s}{D}}\right]^{16}$$

$$\text{and}\quad B=\left[\frac{37'530}{\mathrm{Re}}\right]^{16}$$

Specific Hydraulic Energy

Specific Energy Losses : Singular Losses



- Turbulent Mixing
  - Sudden Expansion
  - Wake
  - Flow Separation
  - etc…

→ For the full list and their calculation refer to pages 35-38 of the hand-out on Moodle

$$gH_{r1\div 2} \approx \left(1-\frac{A_1}{A_2}\right)^2 \frac{C_1^2}{2} = \left(1-\frac{A_2}{A_1}\right)^2 \frac{C_2^2}{2} = \left(\frac{\delta A}{A_2-\delta A}\right)^2 \frac{C_2^2}{2}$$

Specific Hydraulic Energy

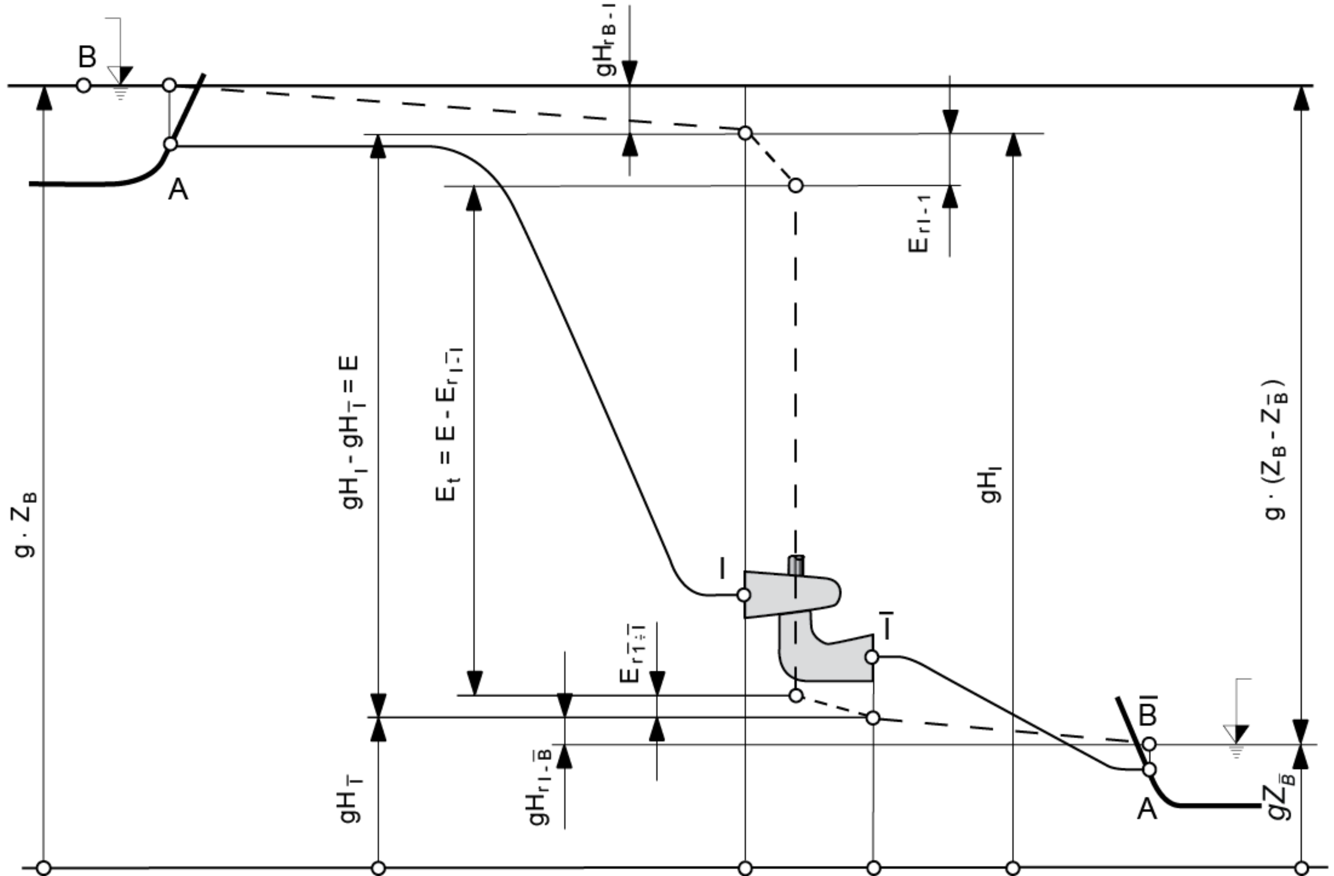
Specific Energy Losses : Total Losses

- Distributed Losses : $gH_{r1\div2} = \lambda\left(Re;\frac{k}{D_h}\right)\frac{16}{\pi^2}\frac{L_{\widehat{12}}}{D_h^5}\frac{Q^2}{2} \quad \left(\mathrm{J}\cdot\mathrm{kg}^{-1}\right)$
- Singular Losses for the ith Component $gH_{ri} = K_i\frac{1}{A_i^2}\frac{Q^2}{2} \quad \left(\mathrm{J}\cdot\mathrm{kg}^{-1}\right)$
- General Formula $gH_{rx\div y} = \sum_{i=1}^{n} K_i\frac{1}{A_i^2}\times\frac{Q^2}{2} \quad \left(\mathrm{J}\cdot\mathrm{kg}^{-1}\right)$

$$= \underbrace{\sum_{i=1}^{n} K_i\frac{A_{ref}^2}{A_i^2}}_{K_{Inst.}}\times\frac{Q^2}{2A_{ref}^2} = K_{Inst.}\times\frac{Q^2}{2A_{ref}^2}$$

Specific Hydraulic Energy

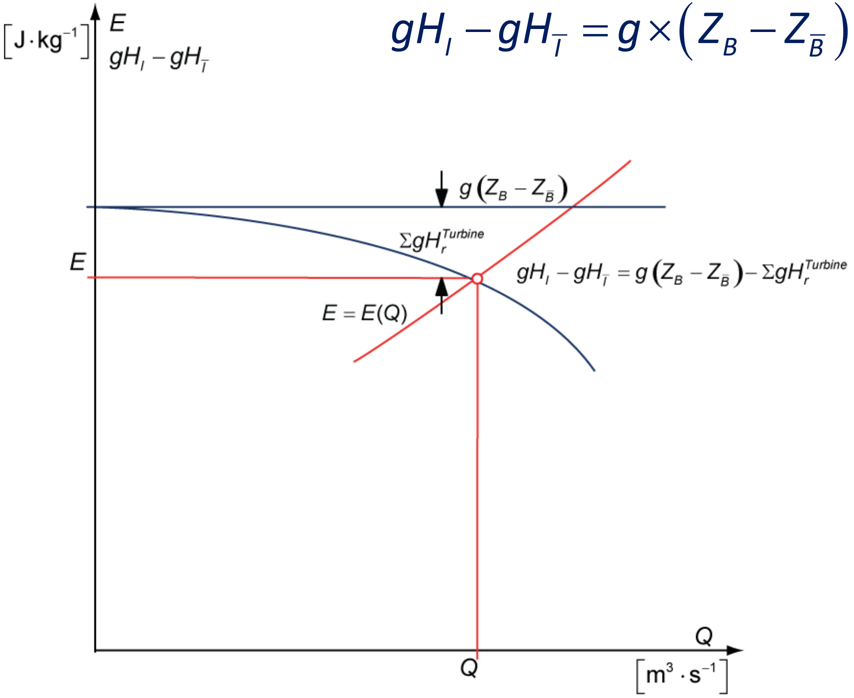
Specific Energy Balance: generating mode



Specific Hydraulic Energy

Hydraulic Characteristics : generating mode

$$gH_I - gH_{\bar{I}} = g \times \left(Z_B - Z_{\bar{B}}\right) - \sum gH_r$$



Specific Hydraulic Energy

Hydraulic Drive : generating mode

- Machine Power Output

  $$P = \vec{\omega} \cdot \vec{T} \quad (\mathrm{W})$$

- Available Hydraulic Power

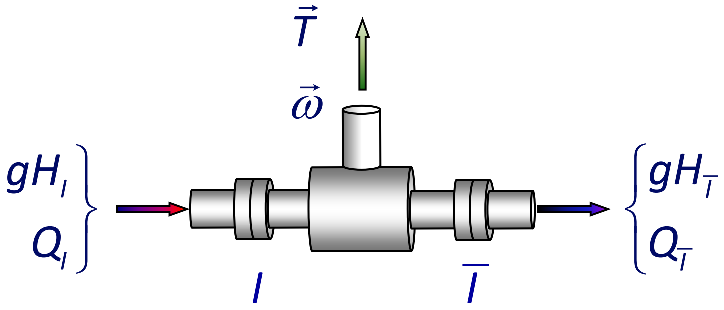
  $$\underbrace{P_h}_{\frac{\mathrm{J}}{\mathrm{s}}} = \underbrace{\rho Q}_{\frac{\mathrm{kg}}{\mathrm{m}^3} \times \frac{\mathrm{m}^3}{\mathrm{s}}} \times \underbrace{E}_{\frac{\mathrm{J}}{\mathrm{kg}}} \quad (\mathrm{W})$$

- Turbine Efficiency

  $$P = \eta^T \times P_h \quad ; \eta^T \leq 100\ \%$$

- Driving Power Defined as Positive :

  $$P \geq 0$$



$$E \triangleq gH_I - gH_{\bar{I}} \quad (\mathrm{J \cdot kg^{-1}})$$

$$P_h = \rho Q \times E$$

$$= \rho Q \times (gH_I - gH_{\bar{I}})$$

Specific Hydraulic Energy
Rotating Train Dynamics

- Rotating train angular momentum equation: $J \times \frac{d\omega}{dt} = T + T_{el} \quad (\mathrm{N \cdot m})$
  - Inertia: J
  - Angular speed: ω
  - Synchronous conditions : $T = -T_{el}$
  - Load rejection : runaway speed $T_{él.} = 0 \Rightarrow \frac{d\omega}{dt} = T > 0$
- Synchronous speed relation :
  - Grid frequency $f_{grid} = 16\tfrac{2}{3}$ Hz; 50 Hz; 60 Hz
  - Number of poles of the synchronous generator z_p
  - Rotating frequency $n = \frac{2 \times f_{grid}}{z_p} \quad (\mathrm{Hz})$

Example

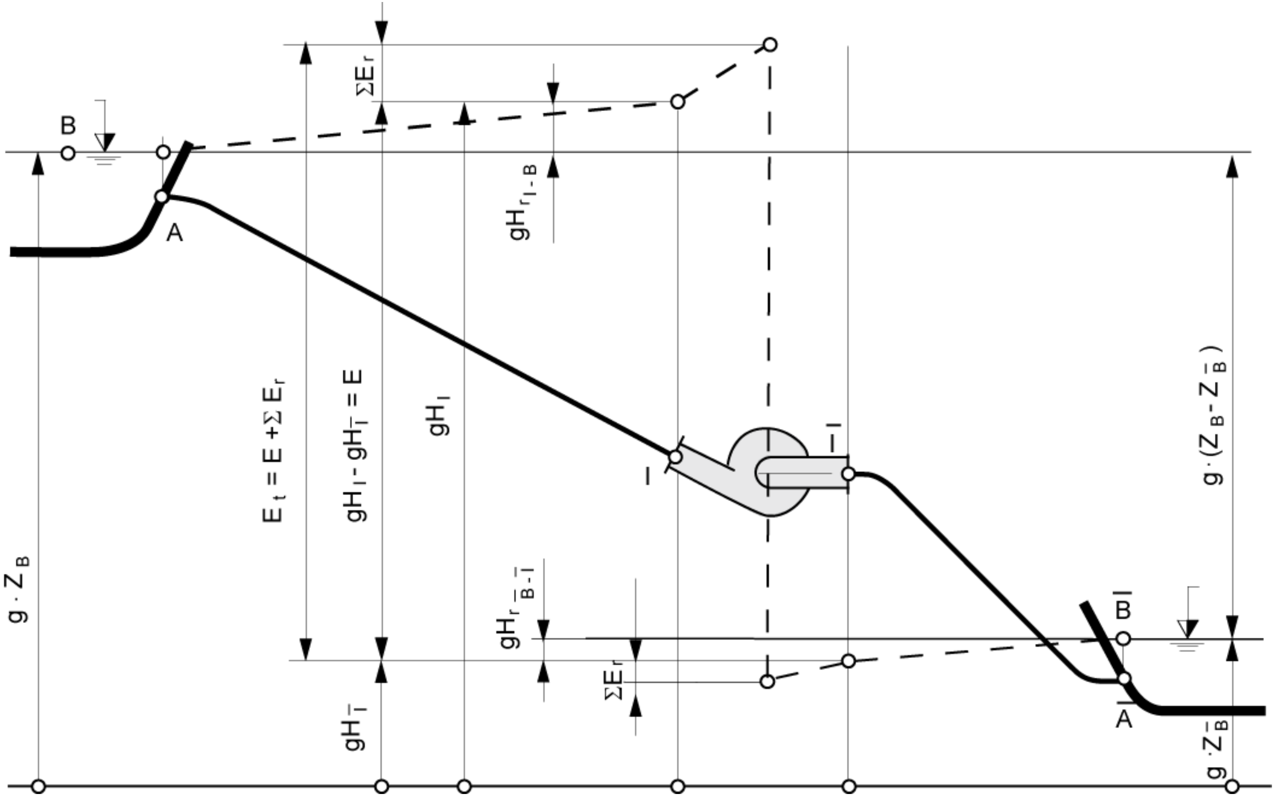
Itaipu (Brasil) rotating train characteristics

- For a discharge of Hydraulic Power:
  $Q = 677 \text{ m}^3 \cdot \text{s}^{-1}$
  $P_h = 786 \text{ MW}$
- For 5% less discharge
  $P_h = 748 \text{ MW}$

- For the Brazilian units number of poles
  $f_{grid} = 60 \text{ Hz}; N = 92.3 \text{ min}^{-1}$
  $z_p = 78$
- For the Paraguayan units number of poles
  $f_{grid} = 50 \text{ Hz}; N = 90.9 \text{ min}^{-1}$
  $z_p = 66$

Specific Hydraulic Energy

Specific Energy Balance: pumping mode



Specific Hydraulic Energy

Specific Energy Balance: pumping mode

- Low Energy Side - Tailwater Side $$gH_{\bar{B}} = \frac{p_{atm}}{\rho} + gZ_{\bar{B}} + 0$$

$$gH_{\bar{B}} = gH_{\bar{I}} + \sum_{\text{Tailwater Side}} gH_r$$

- High Energy Side-Headwater $$gH_I = gH_B + \sum_{\text{Headwater Side}} gH_r$$

$$gH_B = \frac{p_{atm}}{\rho} + gZ_B + 0$$

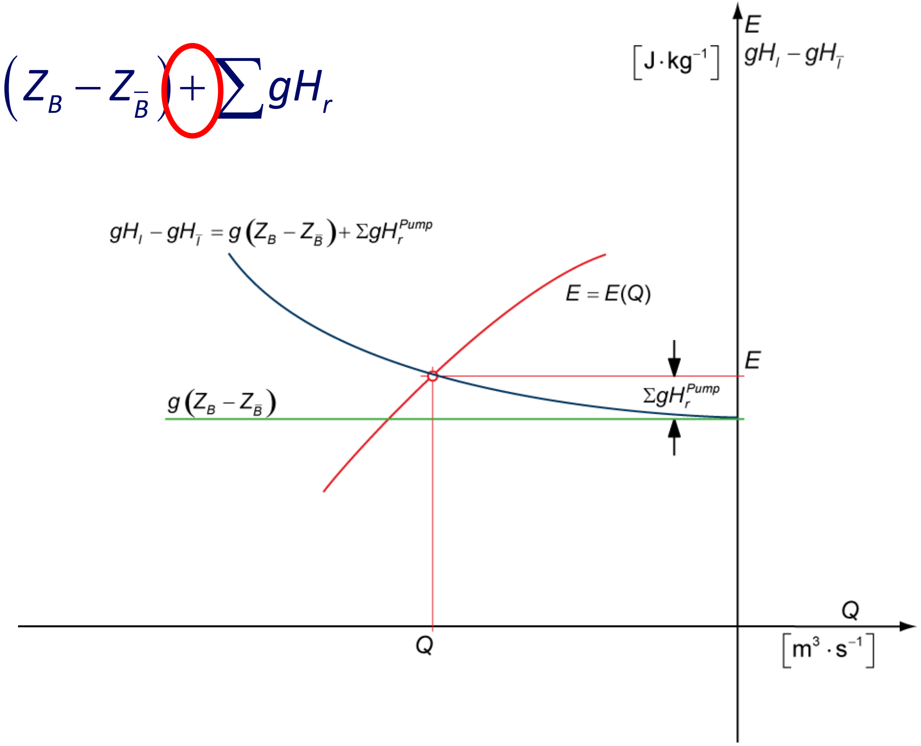
- Specific Energy Supplied by the Pump

$$gH_I - gH_{\bar{I}} = g \times \left(Z_B - Z_{\bar{B}} \right) + \sum gH_r$$

Specific Hydraulic Energy

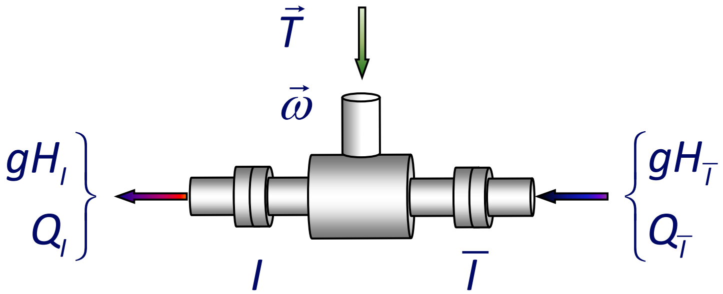
Hydraulic Characteristics: pumping mode

$$gH_I - gH_{\bar{I}} = g\left(Z_B - Z_{\bar{B}}\right) + \sum gH_r$$



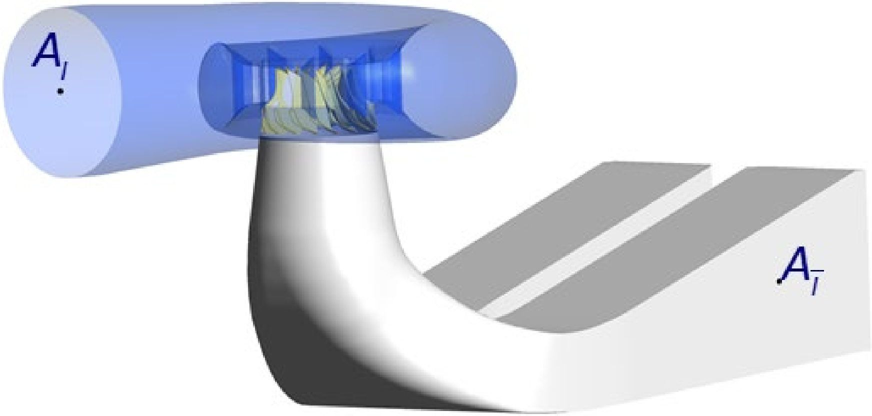
Specific Hydraulic Energy

Hydraulic Brake : pumping mode



- Power input defined as negative $P = \vec{\omega} \cdot \vec{T} < 0$
- Pump Specific Energy $E \triangleq gH_I - gH_{\bar{I}} \geq 0 \quad \left(\mathrm{J \cdot kg^{-1}}\right)$
- Discharge defined as negative
- Hydraulic Power Output $P_h = \rho \times Q \times E \quad (\mathrm{W})$
- Power Input P
- Pump Efficiency $\eta^P = \dfrac{P_h}{P}$

Power Exchange



- Hydraulic Power

$$P_h = P_I - P_{\bar{I}} \quad (\mathrm{W})$$

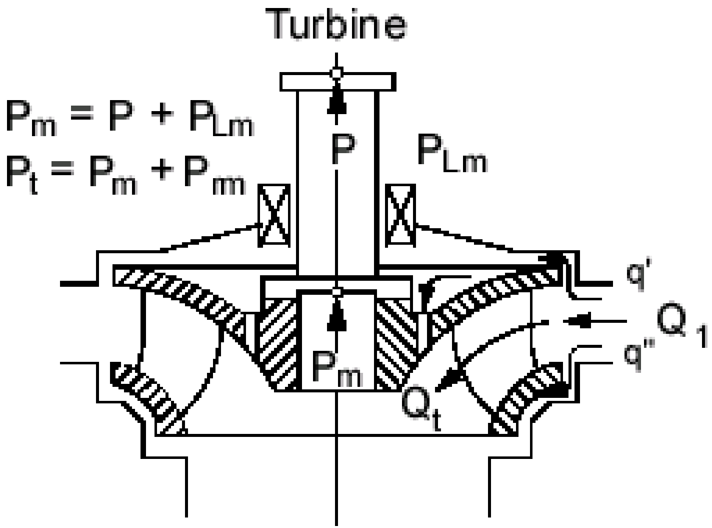
- Work-Generating:
  - Turbine

$$\eta^T P_h = P > 0$$

- Work-Absorbing
  - Pump

$$P_h = \eta^P P < 0$$

Turbine Mechanical Power



$$P_m = \underbrace{P}_{\text{Machine Power}} + \underbrace{P_{Lm}}_{\text{External Mechanical Power Losses}}$$

$$P_t = \underbrace{P_m}_{\text{Runner Mechanical Power}} + \underbrace{P_{rm}}_{\text{Internal Mechanical Power Losses}}$$

$$P_h = \underbrace{P_t}_{\text{Extracted Power}} + \underbrace{\sum P_r}_{\text{Flow Power Dissipation}} + \underbrace{\sum P_q}_{\text{Leakage Flow Power}}$$

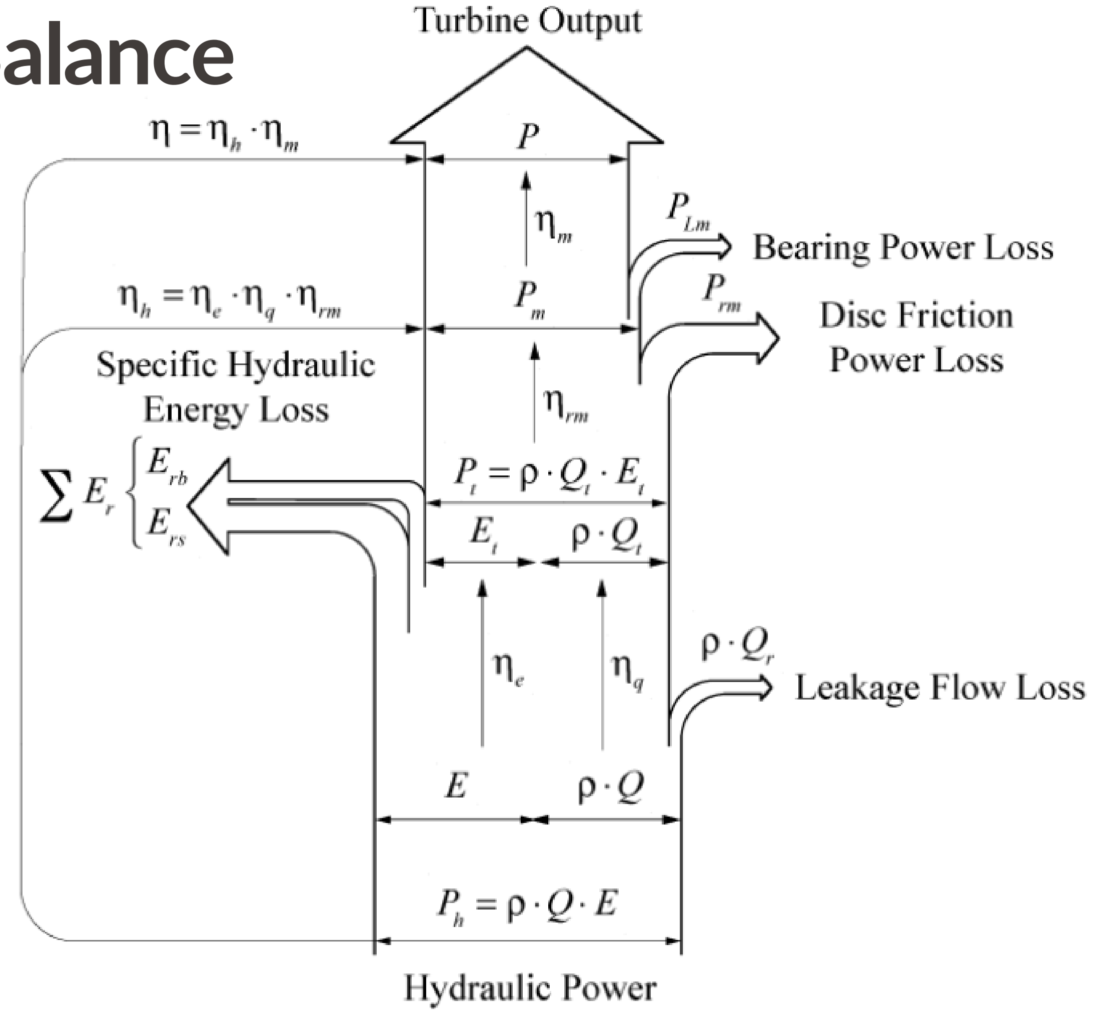
Turbine Power Balance

- Extracted Energy

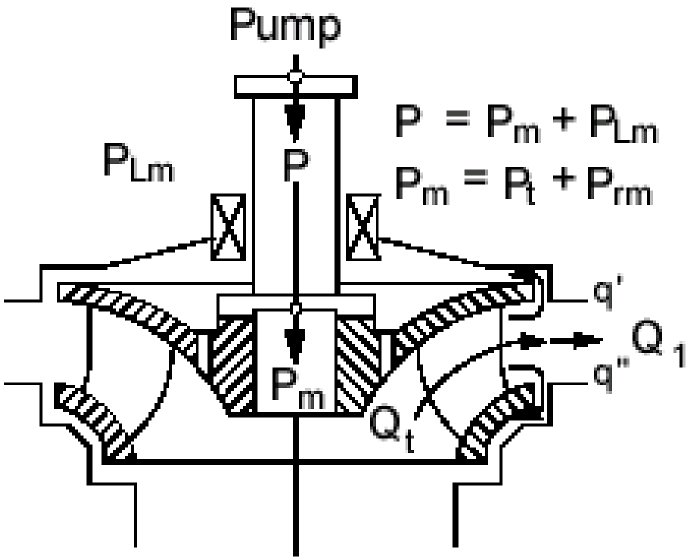
$$E_t = \frac{P_t}{\rho Q_t}$$

- Driving Torque

$$P_t = \vec{\omega} \cdot \vec{T}_t$$



Pump Mechanical Power



$$\overbrace{P}^{\text{Machine Power}} = P_m + \underbrace{P_{Lm}}_{\substack{\text{External Mechanical}\\ \text{Power Losses}}}$$

$$\overbrace{P_m}^{\substack{\text{Impeller Mechanical}\\ \text{Power}}} = P_t + \underbrace{P_{rm}}_{\substack{\text{Internal Mechanical}\\ \text{Power Losses}}}$$

$$\overbrace{P_t}^{\text{Supplied Power}} = P_h + \underbrace{\sum P_r}_{\substack{\text{Flow Power}\\ \text{Dissipation}}} + \underbrace{\sum P_q}_{\substack{\text{Leakage Flow}\\ \text{Power}}}$$

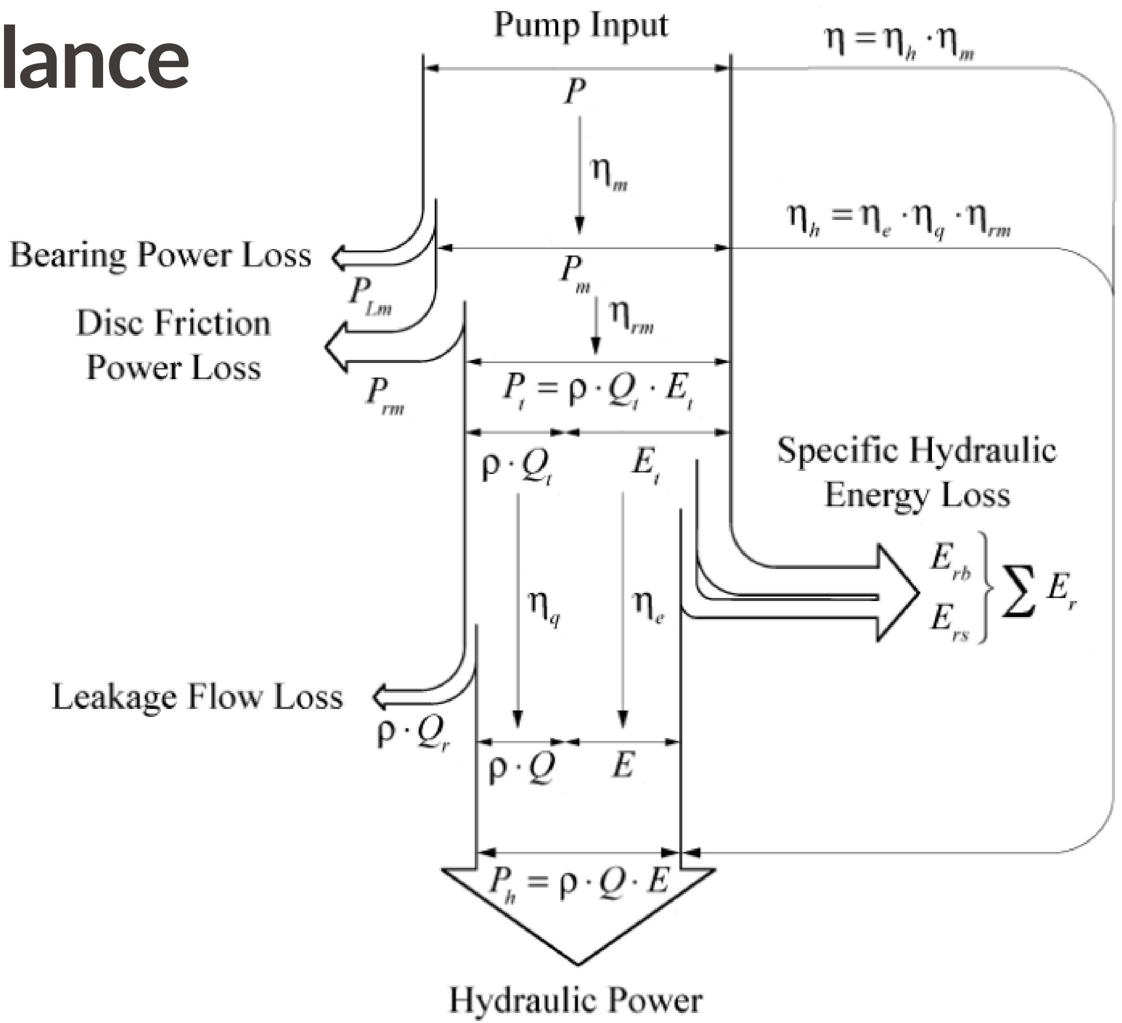
Pump Power Balance

- Supplied Energy

$$E_t = \frac{P_t}{\rho Q_t}$$

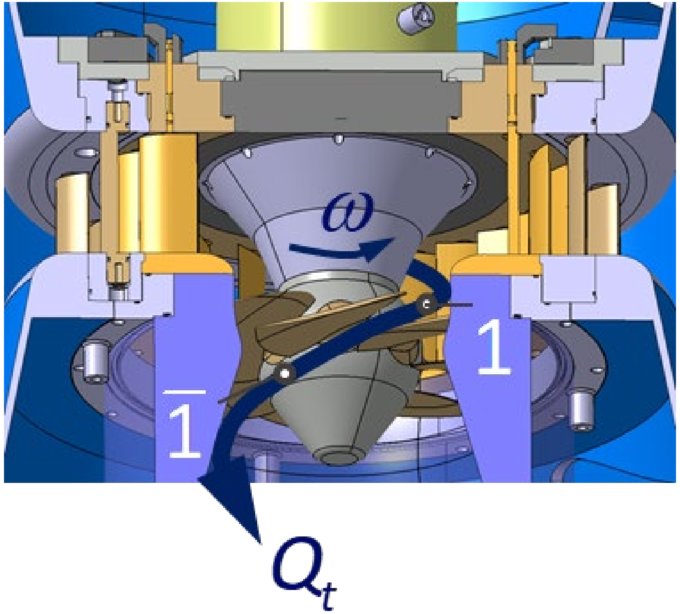
- Resisting Torque

$$P_t = \vec{\omega} \cdot \vec{T}_t$$



Classification of Hydraulic Runners

Runner/impeller specific energy transfer



Brillant Extension Project, British Columbia, Canada, Kaplan Turbine CAD Model, PF2 EPFL Test Rig

- Traversing Discharge

  $Q_t \quad \left(\mathrm{m^3 \cdot s^{-1}}\right)$

- Transferred Specific Energy

  $gH_1 - gH_{\bar{1}} = E_t \pm E_{rb} \quad \left(\mathrm{J \cdot kg^{-1}}\right)$

- Power Transfer

  $P_t = \rho Q_t E_t \quad (\mathrm{W})$

  - Drive: Turbines $P > 0$
  - Brake: Pumps, Propellers $P < 0$

Classification of Hydraulic Runners

Runner/impeller specific energy transfer

- Transferred Specific Energy

$$gH_1 - gH_{\bar{1}} = E_t \pm E_{rb} \quad \left(\mathrm{J \cdot kg^{-1}}\right)$$

- Specific Energy

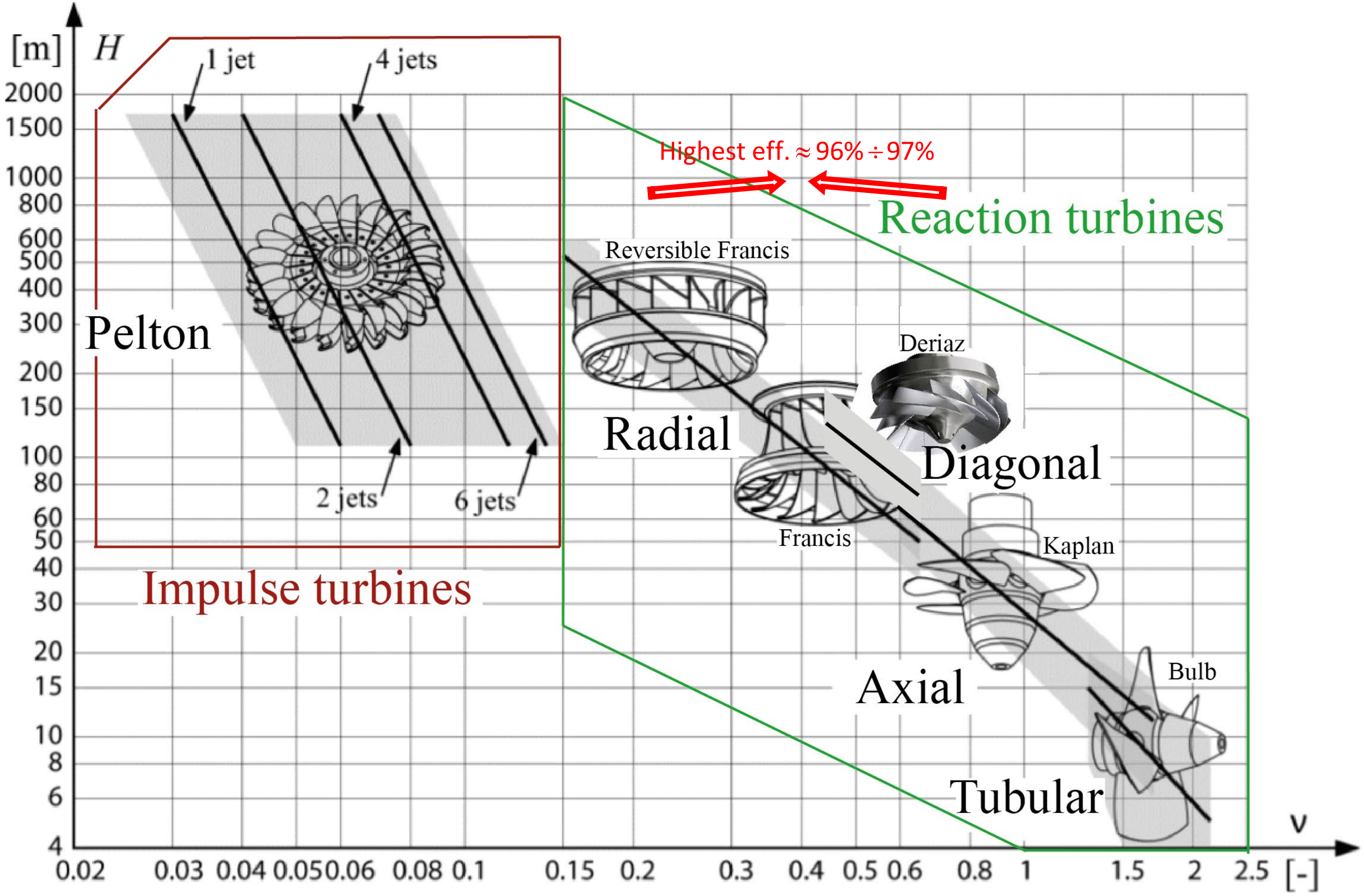
$$gH = \frac{p}{\rho} + gZ + \frac{C^2}{2} \quad \left(\mathrm{J \cdot kg^{-1}}\right)$$

- Specific Energy Balance:

$$E_t = \overbrace{\underbrace{\left(\frac{p_1}{\rho} - \frac{p_{\bar{1}}}{\rho}\right)}_{\text{Displacement}} + \underbrace{\left(\frac{C_1^2}{2} - \frac{C_{\bar{1}}^2}{2}\right)}_{\text{Impulse}}}^{\text{Reaction}} + \underbrace{\left[gZ_1 - gZ_{\bar{1}}\right]}_{\text{Water Wheel}} \pm \underbrace{E_{rb}}_{\text{Loss}}$$

$$\left(\mathrm{J \cdot kg^{-1}}\right)$$

Classification of Hydraulic Runners



Head = H (m)

Discharge = Q $(m^3 \cdot s^{-1})$

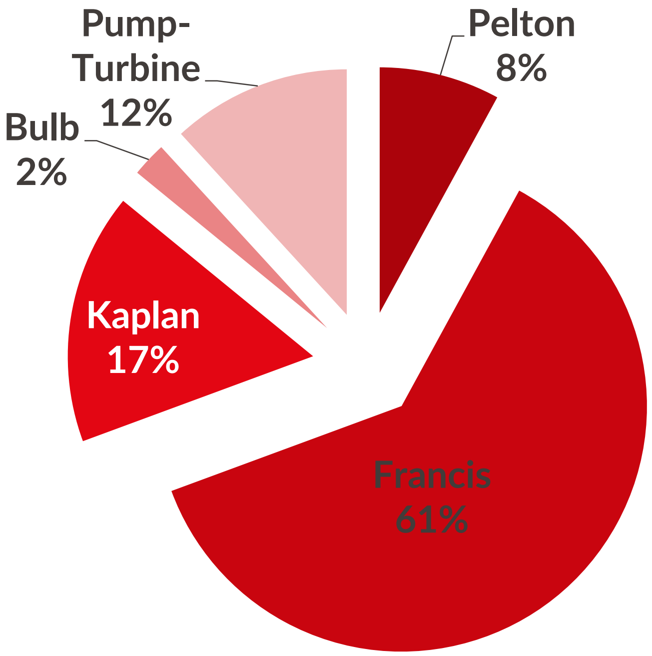
Speed = N (min^{-1})

$$\nu = 2^{\frac{1}{4}} \pi^{\frac{1}{2}} \times n \times \frac{Q^{\frac{1}{2}}}{E^{\frac{3}{4}}}$$

Classification of Hydraulic Runners

International market in 2010

- Breakdown by Types of Turbines
  - 1’038 GW Installed Capacity in 2010
  - 1’000 GW to be built before 2050 Greenfields Project



Classification of Hydraulic Runners
Specific Speed

Definition of dimensionless numbers for the specific energy conversion:

- "Power Plant" Conditions
  - Discharge $[Q] = L^3T^{-1} \cdots (\mathrm{m}^3 \cdot \mathrm{s}^{-1})$
  - Specific Energy $\left[E \triangleq gH_I - gH_{\bar{I}}\right] = L^2T^{-2} = \dfrac{ML^2T^{-2}}{M} \cdots (\mathrm{J} \cdot \mathrm{kg}^{-1})$
- Unit Characteristic
  - Angular Speed $[\omega] = T^{-1} \cdots (\mathrm{s}^{-1})$
  - Turbine/Pump Dimension $[D] = L \cdots (\mathrm{m})$

...

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Classification of Hydraulic Runners

Specific Speed

- Dimensionless Angular Speed Condition $[\nu]=\left[\omega\times Q^{\alpha}\times E^{\beta}\right]=M^{0}\times T^{0}\times L^{0}$

$$T^{0}\times L^{0}=T^{-1}\times L^{3\alpha}T^{-\alpha}\times L^{2\beta}T^{-2\beta}$$

- Yields to Solve the Linear System
  - Dimension of Time $-1-\alpha-2\beta=0$
  - Dimension of Length $3\alpha+2\beta=0$
- Solution $\alpha=\dfrac{1}{2};\quad \beta=\dfrac{-3}{4}$

Classification of Hydraulic Runners

Specific Speed

- Discharge Coefficient (Dimensionless) $\varphi = \frac{Cm}{U}$

- Specific Energy Coefficient (Dimensionless) $\psi = \frac{E}{\frac{U^2}{2}}$

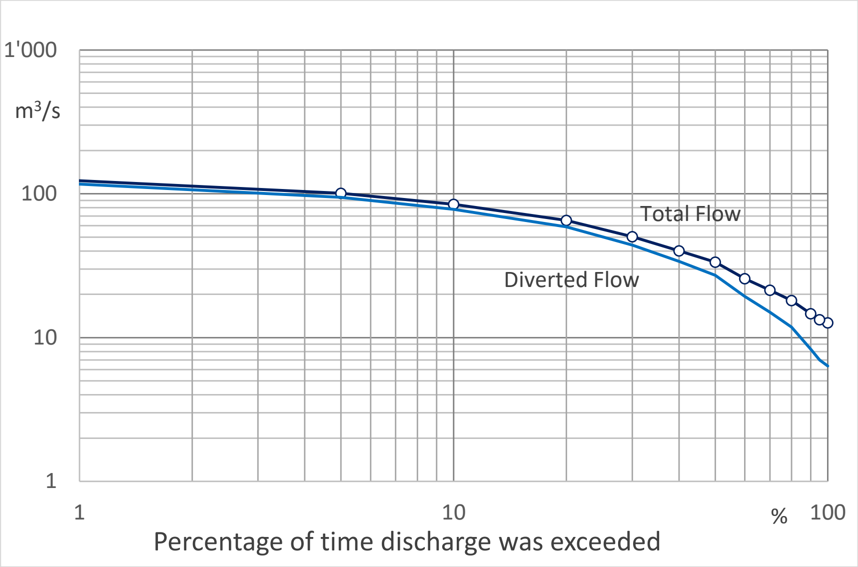
- Specific Speed $\nu = \frac{\varphi^{\frac{1}{2}}}{\psi^{\frac{3}{4}}} = \frac{\omega}{\pi^{\frac{1}{2}} 2^{\frac{3}{4}}} \times \frac{|Q|^{\frac{1}{2}}}{E^{\frac{3}{4}}}$

Classification of Hydraulic Runners

Matching Turbine Specific Speed to Site Condition

- Site Potential Specific Energy $g\left(Z_B - Z_{\overline{B}}\right)$
  - Available Specific Energy $E = gH_I - gH_{\overline{I}} = \left(g_B Z_B - g_{\overline{B}} Z_{\overline{B}}\right) - \sum gH_r \quad \left(\mathrm{J \cdot kg^{-1}}\right)$
- Site Flow Duration Statistics
  - Average Discharge

$Q_{Instal.} \quad \left(\mathrm{m^3 \cdot s^{-1}}\right)$



Classification of Hydraulic Runners

Matching Turbine Specific Speed to Site Condition

- Targeted Unit Specific Speed $\nu = \frac{\omega}{\pi^{\frac{1}{2}} 2^{\frac{3}{4}}} \frac{\left(Q_{instal.} / z_{units} \right)^{\frac{1}{2}}}{E^{\frac{3}{4}}}$
  - Rated Head $H = \frac{E}{g}$
  - Matching number of units z_{units}
  - Matching rotating speed $N = \frac{2 \times f_{grid}}{z_p} \times 60\ \mathrm{s}\ \left(\mathrm{min}^{-1}\right)$
- Check runaway speed
- Check apparent power per poles
  - Air cooling < 28 MVA < Water cooling < 35 MVA

Classification of Hydraulic Runners

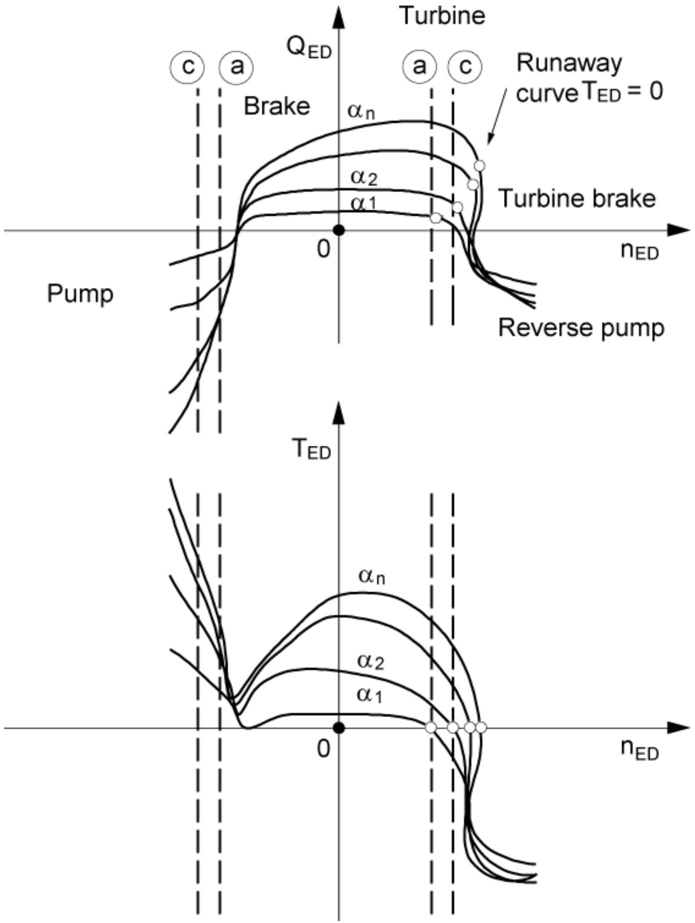
Operating range

- IEC Discharge Factor

$$Q_{ED} = \frac{Q}{D^2 E^{\frac{1}{2}}}$$

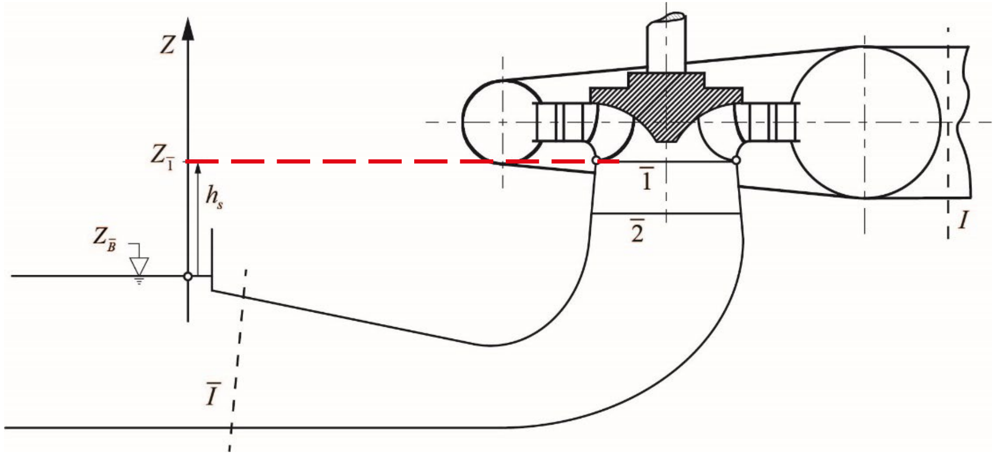
- IEC Speed Factor

$$n_{ED} = \frac{nD}{E^{\frac{1}{2}}}$$



Classification of Hydraulic Runners

Machine setting level



Cut-view of a Francis turbine

- Minimum setting level to avoid cavitation
  - Specific Energy

    $$E \triangleq gH_I - gH_{\bar{I}} > 0 \quad \left(\mathrm{J \cdot kg^{-1}}\right)$$

  - Net Positive Suction Specific Energy

    $$NPSE \triangleq gH_{\bar{I}} - \frac{p_v}{\rho} - gZ_{ref}$$

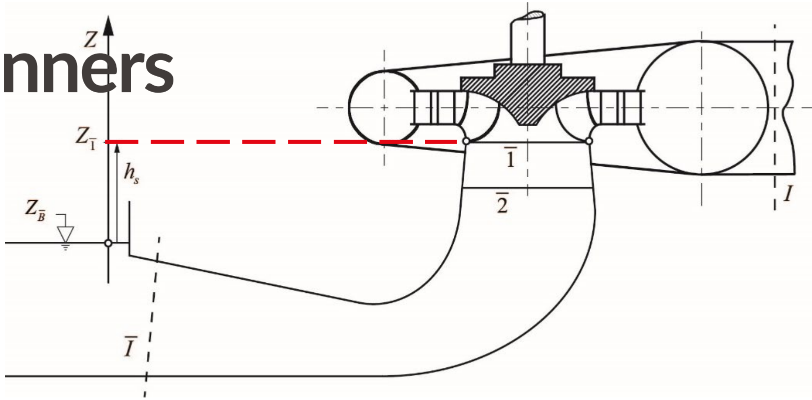
    $$\left(\mathrm{J \cdot kg^{-1}}\right)$$

  - Setting Level

    $$h_s = Z_{ref} - Z_{\bar{B}} = Z_{\bar{1}} - Z_{\bar{B}}$$

Classification of Hydraulic Runners

Machine setting level



- With Respect to the Tail Water Level $\bar{B}$
  - Specific Energy Balance $gH_{\bar{I}} = gH_{\bar{B}} + gH_{r\bar{I} \div \bar{B}}$

$$gH_{\bar{B}} = \frac{p_a}{\rho} + gZ_{\bar{B}}$$

  - For Turbines $gH_r \approx \frac{C_{\bar{I}}^2}{2}$

$$NPSE \approx \frac{p_a}{\rho} - \frac{p_v}{\rho} - gh_s + \frac{C_{\bar{I}}^2}{2} = g \times NPSH$$

  - For Pumps $gH_r \approx \frac{C_{\bar{I}}^2}{2} \approx 0$

$$NPSE \approx \frac{p_a}{\rho} - \frac{p_v}{\rho} - gh_s = g \times NPSH$$

  - Thoma number: $\sigma_{TH} = \frac{NPSE}{E}$

Nomenclature:

$NPSH$ (m) Net Positive Suction Head

Classification of Hydraulic Runners

Machine setting level

Nomenclature:

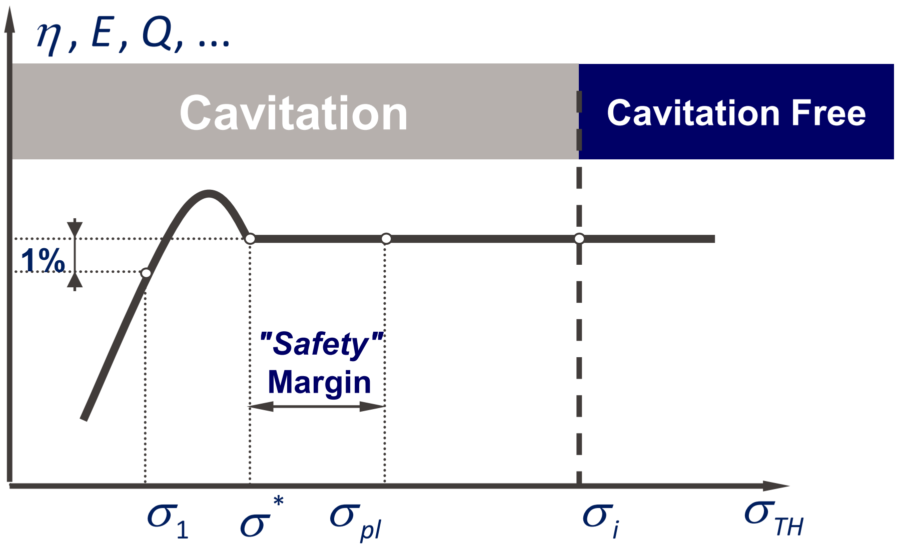
η (-) Efficiency

σ_i (-) Inception: 1st Cavity !

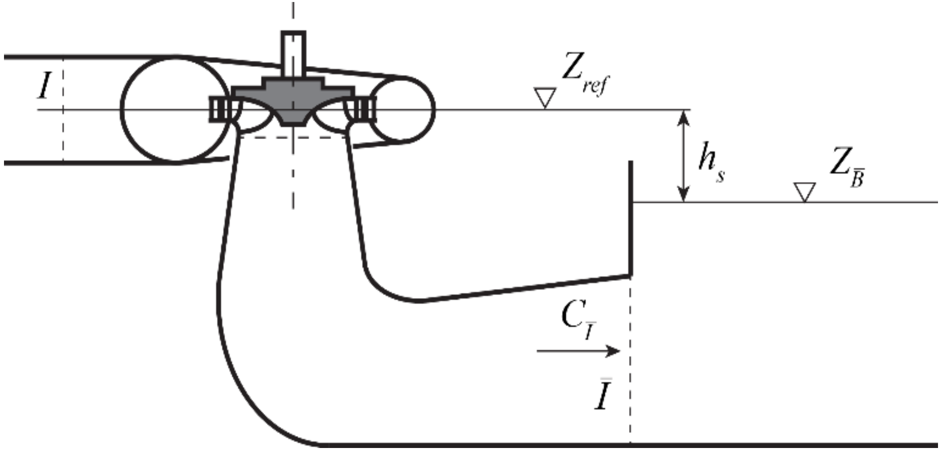
σ_{pl} (-) Plant Value

σ_* (-) 1st Efficiency Change

σ_1 (-) 1% Efficiency Drop



Exercise



Installation of the turbine and setting level

DATA

$C_{\bar{I}} = 0.86 \ \text{m} \cdot \text{s}^{-1}$

$Z_{\bar{B}} = 175.6 \ \text{m}$

Compute Z_{ref} , the setting elevation of the turbine units, to achieve a net positive suction head (NPSH) of 13.4 m. The atmospheric pressure and the saturated vapor pressure can be assumed as $p_{atm} = 1.0$ bar and $p_v = 2343$ Pa, respectively.